$(E_{ijkl} - C_{ijkl})$ presented here is equal to the E used by Lu and Wang (3). This has the advantage that in the absence of quaternary data, the corresponding E_{ijkl} may be set to zero to obtain a better approximation than by setting $(E_{ijkl} - C_{ijkl})$ to zero.

NOTATION

a = interaction coefficient
A = binary two- and three-suffix coefficient
C = ternary coefficient
D = binary four-suffix coefficient
E = quaternary coefficient
ΔG^{xs} = molar excess Gibbs free energy
i, j, k, l, m, n, p = index representing components
n = moles
P = pressure
R = gas constant
T = absolute temperature
x = mole fraction
γ = activity coefficient

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A Generalized Method for Predicting the Minimum Fluidization Velocity

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In a recent article Narsimhan (*18*) presented a generalized expression for the minimum fluidization velocity by extending the correlation proposed by Leva, Shirai, and Wen (*14*) into intermediate and turbulent flow regions. Based on a similar approach by employing the fixed-bed pressure drop equation of Ergun (*7*), an expression for the minimum fluidization velocity quite different from that of Narsimhan has been obtained (*23*).

It is the purpose of this communication to compare these two correlations and to examine the validity and applicability of each.

The generalized expression given by Narsimhan consists of three equations [Equations (6), (9), and (11) in his communication (*18*)].

The correlation obtained by Wen and Yu (*23*) can be represented by

$$(N_{Re})_{mf} = \sqrt{(33.7)^2 + 0.0408\, N_{Ga}} - 33.7 \qquad (1)$$

For nonspherical particles, the particle diameter d_p is defined as the equivalent diameter of a spherical particle with the same volume. As an approximation, the particle diameter may be calculated from the geometric mean of the two consecutive sieve openings without introducing serious errors (*26*).

The major differences between the two correlations are the minimum fluidization voidage ϵ_{mf} and the shape factor ϕ_s.

1. Narsimhan considered that for spherical particles, ϵ_{mf} has the value of 0.35 and is independent of the particle diameter, provided that the wall effect can be neglected. From the literature data (*16, 20, 24*), as well as from the experimental data of the present investigation (*23*), ϵ_{mf} for spherical particles can be shown to vary from 0.36 to 0.46. Different average values of ϵ_{mf} have

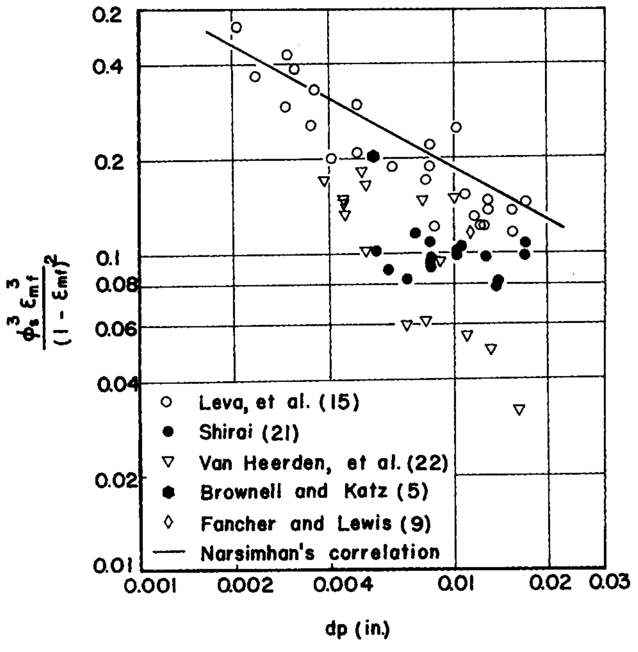

Fig. 1. Correlation of voidage shape factor function $\frac{\phi_s^3 \epsilon_{mf}^3}{(1-\epsilon_{mf})^2}$.

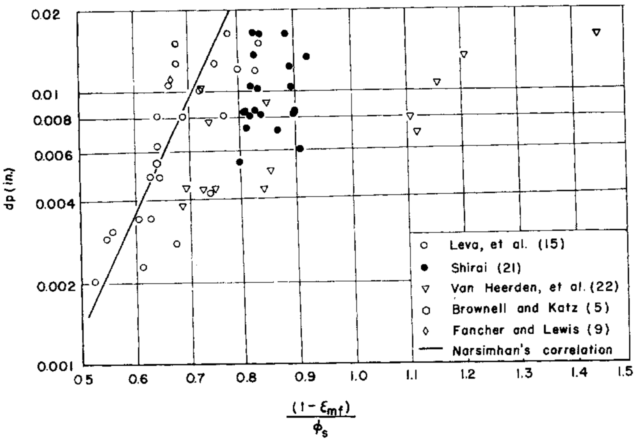

Fig. 2. Correlation of voidage shape factor function $\frac{(1-\epsilon_{mf})}{\phi_s}$.

been reported, such as, 0.386 (*21*), 0.40 (*4*), or 0.42 (*23*). Therefore, the value of 0.35 used by Narsimhan for spheres seems to be too small.

2. For nonspherical particles, Narsimhan considered that ϵ_{mf} depends on the particle diameter if it is less than 0.02 in. On the other hand, if greater than 0.02 in., he considered that ϵ_{mf} is independent of particle diameter.

From Figures 1 and 2, Narsimhan's correlations for small particles are seen to be valid only for a limited number of the experimental data points. Owing to the lack of literature data, the validity of Narsimhan's correlation for large particles cannot be tested. Narsimhan used only seven data points to establish his correlation.

It is believed that if the wall effect can be neglected, ϵ_{mf} should depend only on the shape factor regardless of the diameter of the particle. In Figure 3, ϵ_{mf} are plotted vs. ϕ_s for spherical as well as nonspherical particles. Although some scattering may be observed, a general trend is seen to exist between ϵ_{mf} and ϕ_s. The following two approximate relations are obtained which cover the d_p range from 0.002 to 1.97 in., ϵ_{mf} from 0.385 to 0.935, ϕ_s from 0.136 to 1.0, and with a particle diameter to column diameter ratio ranging from 0.000807 to 0.25.

$$(1 - \epsilon_{mf})/(\phi_s^2\, \epsilon_{mf}^3) \cong 11 \tag{2}$$

$$1/(\phi_s\, \epsilon_{mf}^3) \cong 14 \tag{3}$$

In order to compare the validity of Equation (1) and that of Narsimhan's correlation, the deviations based on

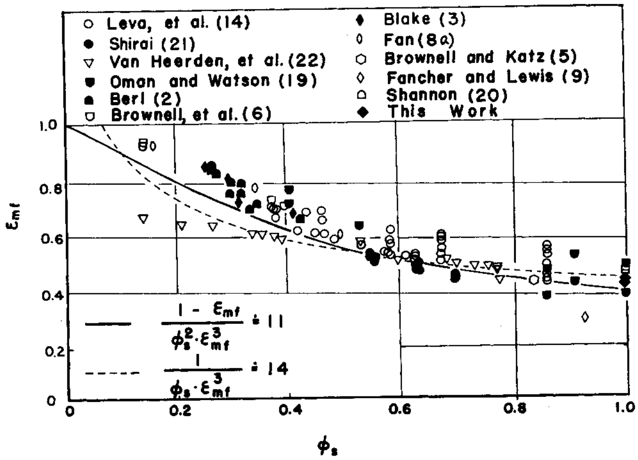

Fig. 3. Relation between ϵ_{mf} and ϕ_s.

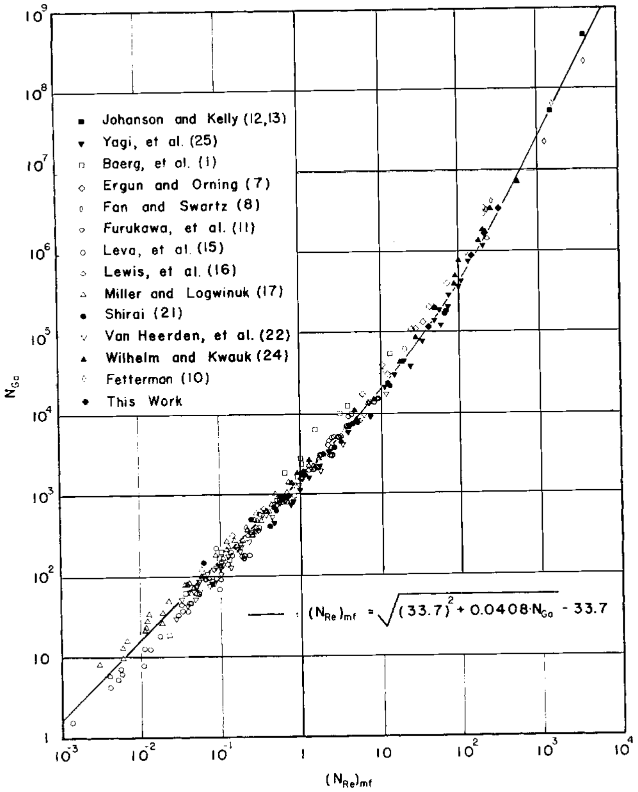

Fig. 4. A generalized correlation for minimum fluidization velocity.

284 points appearing in Figure 4 are computed. The results are given in Table 1. Equation (1) gives an overall standard deviation of 34% and an average deviation of ± 25% based on 284 points available in the literature as shown in Figure 4. An overall standard deviation of 46% and an average deviation of ± 34% are obtained from Narsimhan's correlations based on 267 data points tested. The difference in the number of data points tested for the validity of the two correlations is due to the lack of information pertaining to the shape factor. The proposed correlation represented by a single relation in Equation (1) does not require this information and covers the widest $(N_{Re})_{mf}$ range, from 0.001 to 4000, heretofore attempted.

The advantages of Equation (1) over the Narsimhan's correlations may be summarized as follows: (1) Equation (1) is considerably simpler. (2) Equation (1) gives greater accuracy. (3) Equation (1) may be expressed in a convenient graphical form such as shown in Figure 4 for a rapid estimation of the minimum fluidization velocity.

TABLE 1. COMPARISON OF EQUATION (1) AND NARSIMHAN'S CORRELATIONS

Particle characteristics	No. of data points	Standard deviations based on Narsimhan's correlations, %	Standard deviations based on Equation (1), %
Spherical	55	43.4	34.1
Nonspherical			
$d_p < 0.02$ in.	203	38.6	37.6
$d_p > 0.02$ in.	9	135.3	21.3
Overall standard deviation:		46*	34†

* Calculation based on 267 data points.
† Calculation based on all the 284 data points.

ACKNOWLEDGMENT

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NOTATION

d_p = particle diameter, L
$f(\ )$ = function of ( )
g = acceleration due to gravity, L/θ^2
G_{mf} = minimum fluidization flow rate, $M/L^2\theta$
N_{Ga} = Galileo number = $d_p^3\,\rho_f(\rho_s-\rho_f)g/\mu^2$
N_{Re} = particle Reynolds number = $(d_p\,\rho_f V/\mu)$
$(N_{Re})_{mf}$ = particle Reynolds number at onset of fluidization
N_{Ret} = particle Reynolds number at terminal falling velocity
V = superficial fluid velocity, L/θ
ρ_f = fluid density, M/L^3
ρ_s = particle density, M/L^3
μ = fluid viscosity, $M/L\theta$
ϵ_{mf} = minimum fluidization voidage
ϕ_s = $\dfrac{\text{surface area of sphere having the same volume as particle}}{\text{surface area of particle}}$

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Scale-Up of Residence Time Distributions

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Residence time distributions are valuable for understanding the performance of many continuous flow systems and for formulating mathematical models of such systems. The residence time distribution $Y(X)$ referred to in this communication is defined as: $Y(X)dX$ is the fraction of the inflowing (outflowing) stream which will spend (has spent) a time between X and $X + dX$ in the system. For ideal cases such as plug flow, perfectly stirred vessels, laminar flow, etc., the residence time distributions may be obtained analytically. However, in many cases of practical interest, it is necessary to determine the distributions experimentally. This study investigates the possibility of determining residence time distributions of process systems from experimental tests performed on suitably scaled laboratory models.

CRITERIA FOR MODELS

As early as 1953 the conditions under which residence time distributions of large systems can be predicted from